

INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by
Jane Doe

A dissertation submitted to the faculty of
The University of Utah
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Mathematics

The University of Utah

May 2016

Copyright © Jane Doe 2016

All Rights Reserved

The University of Utah Graduate School

STATEMENT OF DISSERTATION APPROVAL

The dissertation of Jane Doe
has been approved by the following supervisory committee members:

Cornelius Lánczos , Chair(s) 17 Feb 2016
Date Approved

Hans Bethe , Member 17 Feb 2016
Date Approved

Niels Bohr , Member 17 Feb 2016
Date Approved

Max Born , Member 17 Feb 2016
Date Approved

Paul A. M. Dirac , Member 17 Feb 2016
Date Approved

by Petrus Marcus Aurelius Featherstone-Hough , Chair/Dean of
the Department/College/School of Mathematics
and by Alice B. Toklas , Dean of The Graduate School.

For my parents, Alice and Bob.

CONTENTS

ABSTRACT	iii
LIST OF FIGURES	vi
LIST OF TABLES	vii
NOTATION AND SYMBOLS	viii
CHAPTERS	
1. THE FIRST	1
1.1 The first section	1
1.1.1 The first subsection	2
1.1.2 The second subsection	2
1.1.3 The third subsection	2
1.1.3.1 The first subsubsection	2
1.1.3.2 The second subsubsection	2
1.1.3.2.1 The first numbered paragraph	2
1.1.3.2.2 The second numbered paragraph	3
1.2 The second section	3
1.3 The third section	5
1.4 Free software packages	6
1.5 Resizing figures	9
1.6 Summary and conclusions	13
2. THE SECOND	15
3. THE THIRD	16
4. THE FOURTH	17
4.1 More on the topic	17
4.2 Even more on the topic	17
4.3 Summary and conclusions	18
APPENDICES	
A. THE FIRST	19
B. THE SECOND	20
C. THE THIRD	21
REFERENCES	24

LIST OF FIGURES

1.1	The first figure.	2
1.2	The second figure.	3
1.3	The third figure.	8
1.4	The fourth figure (at 50% scale).	10
1.5	The fifth figure (at 75% scale).	10
1.6	The sixth figure (at native size).	10
1.7	The seventh figure (at 125% scale).	10
1.8	The eighth figure (at 175% scale).	10
1.9	The ninth figure (at 50% scale)	12
1.10	The tenth figure (at 75% scale)	12
1.11	Using \LaTeX picture mode	13

LIST OF TABLES

1.1	Lowercase Greek letters.	4
1.2	Uppercase Greek letters.	6

NOTATION AND SYMBOLS

α	fine-structure (dimensionless) constant, approximately $1/137$
α	radiation of doubly-ionized helium ions, He ⁺⁺
β	radiation of electrons
γ	radiation of very high frequency, beyond that of X rays
γ	Euler's constant, approximately $0.577\,215 \dots$
δ	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
ϵ	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
\dots	\dots
$\psi(x)$	logarithmic derivative of the gamma function
ω	frequency

CHAPTER 1

THE FIRST

This is a chapter. Remember that there should *always* be at least of few lines of prose after each sectional heading: failure to do so is a disservice to your readers, and also produces incorrect vertical spacing.

1.1 The first section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.1** on the following page, we have a picture, and the \LaTeX markup to include it looks like this:

```
\begin{figure}[t]
  \centerline{\includegraphics{fig1}}
  \caption{The first figure.}%
  \figlabel{fig1}
\end{figure}
```

We intentionally omitted an extension on the filename, so that this document can be processed with `latex` to get an output `.dvi` file, or with `pdflatex` to get an output `.pdf` file. The first case uses the file `fig1.eps`, and the second uses `fig1.pdf`. The `distill` or `ps2pdf` commands can be used to convert from *Encapsulated PostScriptfiles* to *Portable Document Formatfiles*.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

This is Figure 1

Figure 1.1. The first figure.

1.1.1 The first subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.2 The second subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3 The third subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.1 The first subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2 The second subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2.1 The first numbered paragraph Blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah.

blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah
blah blah blah blah blah blah.

1.1.3.2.2 The second numbered paragraph Blah blah blah blah blah blah blah
blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah
blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah
blah blah blah blah blah blah.

1.2 The second section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.2**, we have another picture.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.



This is Figure 2

Figure 1.2. The second figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Table 1.1**, we show the 24-character lowercase Greek alphabet.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.1. Lowercase Greek letters.

α	alpha
β	beta
γ	gamma
δ	delta
ϵ, ε	epsilon
ζ	zeta
η	eta
θ, ϑ	theta
ι	iota
κ	kappa
λ	lambda
μ	mu
ν	nu
ξ	xi
\omicron	omicron
π	pi
ρ	rho
σ, ς	sigma
τ	tau
υ	upsilon
ϕ, φ	phi
χ	chi
ψ	psi
ω	omega

Table 1.2. Uppercase Greek letters. Notice that several have the same letter shapes as Latin letters, and for those, \TeX does not define macro names. For convenience, we supply our own definitions of these macros: `\Alpha`, `\Beta`, `\Epsilon`, `\Zeta`, `\Eta`, `\Iota`, `\Kappa`, `\Mu`, `\Nu`, `\Omicron`, `\Rho`, `\Tau`, and `\Chi`.

A	Alpha
B	Beta
Γ	Gamma
Δ	Delta
E	Epsilon
Z	Zeta
H	Eta
Θ	Theta
I	Iota
K	Kappa
Λ	Lambda
M	Mu
N	Nu
Ξ	Xi
O	Omicron
Π	Pi
ρ	Rho
Σ	Sigma
T	Tau
Y	Upsilon
Φ	Phi
X	Chi
Ψ	Psi
Ω	Omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.4 Free software packages

The Free Software Foundation offers almost 300 software packages, most easily portable to many different operating systems and CPU platforms. They include at least these:

`a2ps`, `acct`, `acm`, `adns`, `alive`, `anubis`, `apl`, `archimedes`, `aris`, `aspell`, `auctex`,
`autoconf-archive`, `autoconf`, `autogen`, `automake`, `avl`, `ballandpaddle`, `barcode`, `bash`,
`bayonne`, `bc`, `binutils`, `bison`, `bool`, `bpel2owfn`, `c-graph`, `ccaudio`, `ccd2cue`, `ccrtp`,

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.5 Resizing figures

In **Figure 1.4** through **Figure 1.8** on the following page, we show how graphics files can be rescaled to convenient sizes, with input like this:

```
\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.5]{fig1}}
  \caption{The fourth figure (at 50\% scale).}%
  \figlabel{fig4}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.75]{fig1}}
  \caption{The fifth figure (at 75\% scale).}%
  \figlabel{fig5}
\end{figure}
```



This is Figure 1

Figure 1.4. The fourth figure (at 50% scale).



This is Figure 1

Figure 1.5. The fifth figure (at 75% scale).



This is Figure 1

Figure 1.6. The sixth figure (at native size).



This is Figure 1

Figure 1.7. The seventh figure (at 125% scale).



This is Figure 1

Figure 1.8. The eighth figure (at 175% scale).

```

\begin{figure}[p]
  \centerline{\includegraphics{fig1}}
  \caption{The sixth figure (at native size).}%
  \figlabel{fig6}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 1.25]{fig1}}
  \caption{The seventh figure (at 125\% scale).}%
  \figlabel{fig7}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 1.75]{fig1}}
  \caption{The eighth figure (at 175\% scale).}%
  \figlabel{fig8}
\end{figure}

```

You can include multiple images, each with its own caption inside a single *unbreakable* figure environment, like this example shown in **Figure 1.9** and **Figure 1.10** on the next page, although you might want to adjust interfigure vertical space with a `\vspace{}` command:

```

\begin{figure}[t]
  \centerline{\includegraphics[scale = 0.5]{fig1}}
  \caption{The fourth figure (at 50\% scale).}%
  \figlabel{fig9}
  \vspace{3ex}
  \centerline{\includegraphics[scale = 0.75]{fig1}}
  \caption{The fifth figure (at 75\% scale).}%
  \figlabel{fig10}
\end{figure}

```

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
 Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah.



This is Figure 1

Figure 1.9. The ninth figure (at 50% scale), boxed with the tenth figure.



This is Figure 1

Figure 1.10. The tenth figure (at 75% scale), boxed with the ninth figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
 blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

As a final example in this chapter, **Figure 1.11** on the following page shows how you can use \LaTeX picture mode for annotating and positioning graphics images prepared outside \LaTeX . The input that produced that figure looks like this:

```
\begin{figure}[t]
  %% The original image is 216bp wide by 72bp high, but we
  %% rescale it to 150 picture units divided by \unitlength:
  %% 150 / 0.75 = 112.5 mm
  \newcommand {\myfig} {\includegraphics[width = 112.5mm]{fig1}}

  \begin{center}
    %% The \unitlength is chosen to make the complete picture fit
    %% within the page margins

    \setlength{\unitlength}{0.75mm}

    %%%      insert (width,height)(lower-left-x,lower-left-y)
    \begin{picture}(170,70)(10,10)
      %% Place the included image FIRST!
      \put(10,10) {\myfig}

      %% Everything that follows OVERLAYS the original image!

      \graphpaper[10](0,0)(170,70)

      %% Mark the image center and corners by centered bullets
      \newcommand {\thedot} {\makebox (0,0) {$\bullet$}}
      \put( 85, 35) {\thedot}
      \put( 10, 10) {\thedot}
```


blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.3 Summary and conclusions

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah.
Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX C

THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in \LaTeX `\cite{...}` commands with their \BibTeX citation labels in reverse alphabetical order, thanks to the `citesort` package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their \BibTeX entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,  
  author = "Albert Einstein",  
  title = "{Eine Neue Bestimmung der Molek{\\"u}ldimensionen}.  
          ({German}) [{A} new determination of molecular  
          dimensions]",  
  type = "Inaugural dissertation",  
  school = "Bern Wyss.",  
  address = "Bern, Switzerland",  
  year = "1905",  
  bibdate = "Fri Dec 17 10:46:57 2004",
```

```

bibsource = "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
note = "Published in \cite{Einstein:1906:NBM}.",
acknowledgement = ack-nhfb,
language = "German",
advisor = "Alfred Kleiner (24 April 1849--3 July 1916)",
URL = "http://en.wikipedia.org/wiki/Alfred_Kleiner",
remark = "Received August 19, 1905 and published February 8,
1906.",
Schilpp-number = "6",
}

```

The `note` field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in `BIBTEX` files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long `note` field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that ..., multiple passes of `LATEX` and `BIBTEX` are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

`BIBTEX` entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes `BIBTEX` entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In `BIBTEX` `title` fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842

by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16, 18]. She was the assistant to Charles Babbage (1791–1871), and they are the world’s first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index (except that this sample thesis doesn’t have one!)

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

REFERENCES

- [1] H. P. BABBAGE, *Babbage: Babbage's analytical engine*, Monthly Notices of the Royal Astronomical Society, 70 (1910), pp. 517–526, 645. Reprinted in [27, §2.3].
- [2] N. H. F. BEEBE AND R. P. C. RODGERS, *<PLOT79>: a comprehensive portable Fortran scientific line graphics system, as applied to biomedical research*, Computers in Biology and Medicine, 19 (1989), pp. 385–402.
- [3] N. H. D. BOHR, *Studier over Metallernes Elektronteori. (Danish) [Studies on the electron theory of metals]*, doktor disputats, Københavns Universitet, København, Danmark, 1911. Afhandling for den filosofiske doktorgrad. [Thesis for the Doctor of Philosophy.]
- [4] W. J. CODY, JR., *Analysis of proposals for the floating-point standard*, Computer, 14 (1981), pp. 63–69.
- [5] E. W. DIJKSTRA, *Functionele beschrijving van de ARRA. (Dutch) [Functional description of the ARRA]*, Tech. Rep. 12, Mathematisch Centrum, Amsterdam, The Netherlands, 1953.
- [6] P. A. M. DIRAC, *Quantum Mechanics*, Ph.D. thesis, Cambridge University, Cambridge, UK, June 1926. According to [12, p. 101], this is the first thesis to be submitted anywhere on the subject of quantum mechanics.
- [7] A. EINSTEIN, *Eine Neue Bestimmung der Moleküldimensionen. (German) [A new determination of molecular dimensions]*, inaugural dissertation, Bern Wyss., Bern, Switzerland, 1905. Published in [8].
- [8] ———, *Eine neue Bestimmung der Moleküldimensionen. (German) [A new determination of molecular dimensions]*, Annalen der Physik (1900) (series 4), 324 (1906), pp. 289–306. See corrections [9, 10]. This is a slightly revised version of Einstein's doctoral dissertation [7].
- [9] A. EINSTEIN, *Bemerkung zu meiner Arbeit: Eine Beziehung zwischen dem elastischen Verhalten. (German) [Remark on my paper: "A relationship between the elastic behavior ..."]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 590–590. See [11].
- [10] A. EINSTEIN, *Berichtigung zu meiner Arbeit: Eine neue Bestimmung der Moleküldimensionen. (German) [Corrections to my work: a new determination of molecular dimensions]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 591–592. See [8].
- [11] ———, *Eine Beziehung zwischen dem elastischen Verhalten und der spezifischen Wärme bei festen Körpern mit einatomigem Molekül. (German) [A relationship between the elastic behavior and the specific heat of solid bodies with monatomic molecules]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 170–174, 590. See remarks [9, 10].

- [12] G. FARMELO, *The Strangest Man: The Hidden Life of Paul Dirac, Mystic of the Atom*, Basic Books, New York, NY, USA, 2009.
- [13] H. H. GOLDSTINE AND A. GOLDSTINE, *The Electronic Numerical Integrator and Computer (ENIAC)*, *Mathematical Tables and Other Aids to Computation*, 2 (1946), pp. 97–110. Reprinted in [27, §7.7].
- [14] P. HALL AND P. PATIL, *Properties of nonparametric estimators of autocovariance for stationary random fields*, *Probability Theory and Related Fields*, 99 (1994), pp. 399–424.
- [15] J. L. HEILBRON AND T. S. KUHN, *The genesis of the Bohr atom*, *Historical Studies in the Physical Sciences*, 1 (1969), pp. vi, 211–290.
- [16] V. R. HUSKEY AND H. D. HUSKEY, *Lady Lovelace and Charles Babbage*, *Annals of the History of Computing*, 2 (1980), pp. 299–329.
- [17] S. C. JOHNSON AND M. E. LESK, *Language development tools*, *The Bell System Technical Journal*, 57 (1978), pp. 2155–2176.
- [18] E. E. KIM AND B. A. TOOLE, *Ada and the first computer: The collaboration between ada, countess of lovelace, and computer pioneer Charles Babbage resulted in a landmark publication that described how to program the world's first computer*, *Scientific American*, 280 (1999), pp. 76–81.
- [19] D. E. KNUTH, *The T_EXbook*, vol. A of *Computers and Typesetting*, Addison-Wesley, Reading, MA, USA, 1986.
- [20] ———, *The METAFONTbook*, vol. C of *Computers and Typesetting*, Addison-Wesley, Reading, MA, USA, 1986.
- [21] ———, *Digital Typography*, CSLI Publications, Stanford, CA, USA, 1999.
- [22] S. N. LAHIRI, Y. LEE, AND N. CRESSIE, *On asymptotic distribution and asymptotic efficiency of least squares estimators of spatial variogram parameters*, *Journal of Statistical Planning and Inference*, 103 (2002), pp. 65–85.
- [23] L. LAMPORT, *L^AT_EX—A Document Preparation System—User's Guide and Reference Manual*, Addison-Wesley, Reading, MA, USA, 1985.
- [24] F. MITTELBACH, M. GOOSSENS, J. BRAAMS, D. CARLISLE, C. ROWLEY, C. DETIG, AND J. SCHROD, *The L^AT_EX Companion*, *Tools and Techniques for Computer Typesetting*, Addison-Wesley, Reading, MA, USA, second ed., 2004.
- [25] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT, AND C. W. CLARK, eds., *NIST Handbook of Mathematical Functions*, Cambridge University Press, Cambridge, UK, 2010.
- [26] POSTSCRIPT DEVELOPER TOOLS & STRATEGIES GROUP, ADOBE SYSTEMS INC., *Adobe font metric files specification — Version 3.0*, Mountain View, CA, USA, Mar. 1990.
- [27] B. RANDELL, ed., *The Origins of Digital Computers: Selected Papers*, *Texts and monographs in computer science*, Springer-Verlag, Berlin, Germany / Heidelberg, Germany / London, UK / etc., third ed., 1982.

- [28] A. ROBBINS AND N. H. F. BEEBE, *Classic Shell Scripting*, O'Reilly Media, Inc., 1005 Gravenstein Highway North, Sebastopol, CA 95472, USA, 2005.
- [29] D. SALOMON, *The Advanced T_EXbook*, Springer-Verlag, Berlin, Germany / Heidelberg, Germany / London, UK / etc., 1995.
- [30] S. SINGH, *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*, Walker and Company, 435 Hudson Street, New York, NY 10014, USA, 1997.
- [31] R. TAYLOR AND A. WILES, *Ring-theoretic properties of certain Hecke algebras*, *Annals of Mathematics*, 142 (1995), pp. 553–572. This paper is a companion to [32], providing the remedy for the flaw in Wiles' 1993 proof of Fermat's Last Theorem. See also [30].
- [32] A. WILES, *Modular elliptic curves and Fermat's Last Theorem*, *Annals of Mathematics*, 142 (1995), pp. 443–551. This paper contains the bulk of the author's proof of the Taniyama–Shimura conjecture and Fermat's Last Theorem, carried out at Princeton University. The companion paper [31] contains the solution to the flaw discovered in the proof that Wiles announced on June 23, 1993, in Cambridge, England. See also [30]. In March 2014, now Royal Society Research Professor Sir Andrew John Wiles of Oxford University was awarded the prestigious Abel Prize in Mathematics for this proof — an award that also carries a cash prize of six million Norwegian crowns, or about US\$722,000.