Sample Exam (96 points)
September 26, 2019

Name: ______________________________

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

Math mode is used to display mathematical content in LaTeX, and there are two main forms of math mode: *display mode* and *inline mode*. Question 1 uses *display mode*, which centers the math content on its own line. Question 2 uses *inline mode* to render the math content within a line of text.

1. (10 points) Find an equation for the tangent line to the following curve at the point (0,1).

$$2xy^3 + y^4 = 1 + x^3y$$

2. (10 points) Use the linearization of $f(x) = \sqrt[3]{x}$ at $x = 8$ to approximate $\sqrt[3]{8.24}$.

3. Evaluate each expression.
   (a) (5 points) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

   (b) (5 points) $\displaystyle\frac{d}{dx}\left[\frac{\sin^2(\pi x)}{\sqrt{3x+1}}\right]$

   (c) (5 points) $\displaystyle\int_1^{e^2} t\ln t\ dt$

4. Using the *displaystyle* command (as in each part of question 3) forces fractions, limits, integrals, etc. to be displayed larger and more clearly even when using inline mode. Without *displaystyle*, those math expressions will be shrunk when using inline mode, like so:
   (a) (5 points) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

   (b) (5 points) $\frac{d}{dx}\left[\frac{\sin^2(\pi x)}{\sqrt{3x+1}}\right]$

   (c) (5 points) $\int_1^{e^2} t\ln t\ dt$

5. (8 points) Use the Gauss-Jordan elimination method to find all solutions (if any) to the following system of equations.

$$\begin{cases} \frac{1}{2}x + \frac{3}{2}y - 2z = 10 \\ 2x + 2y + 4z = 24 \\ x + 2y \qquad = 16 \end{cases}$$

$$\frac{1}{2}x + \frac{3}{2}y - 2z = 10$$
$$2x + 2y + 4z = 24$$
$$x + 2y = 16$$

6. (4 points) Use the following matrices to calculate $(A + 4B)C$.

$$A = \begin{bmatrix} 3 & 4 \\ -5 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & -6 & 2 \\ 1 & 1 & -2 \end{bmatrix}.$$

7. (3 points) The lines $2x - 6y = 3$ and $x + 3y = 9$ are ...
   A. parallel
   B. perpendicular
   C. neither A nor B

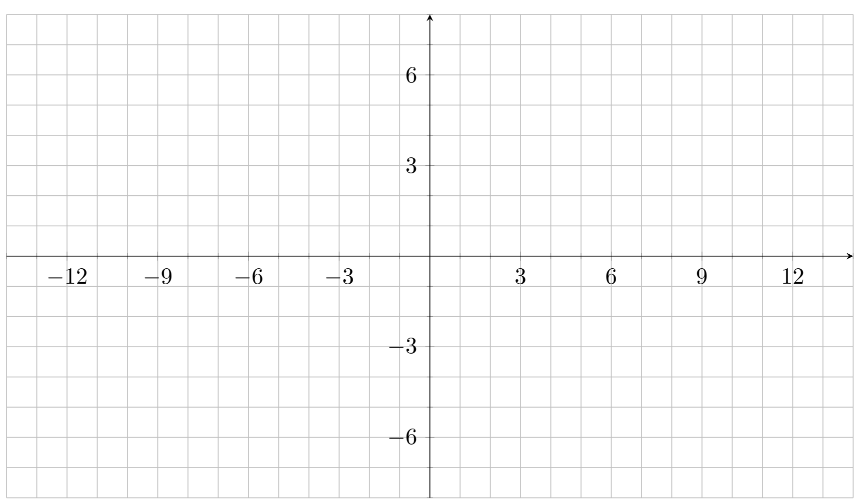
8. (3 points) Which of the following expressions are equivalent to $\ln(16)$? Circle all that apply.
   A. $\ln(20) - \ln(4)$ B. $\ln(\frac{1}{2}) + \ln(32)$ C. $\ln(2) \cdot \ln(8)$ D. $2\ln(4)$

9. (2 points) If $f''(x)$ is ____________________ on some interval, then the graph of f is concave down on that interval.

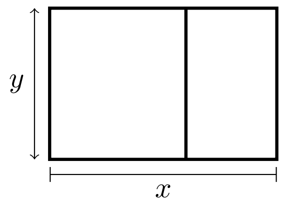
10. (2 points) What is the name for the set of all valid inputs of a function?

10. ____________

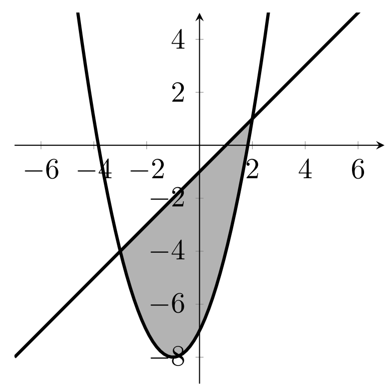
11. (4 points) Sketch the graph of the piecewise function $f(x) = \begin{cases} 5 - x^2 & \text{if } x < 3 \\ x - 2 & \text{if } x \geq 3 \end{cases}$.



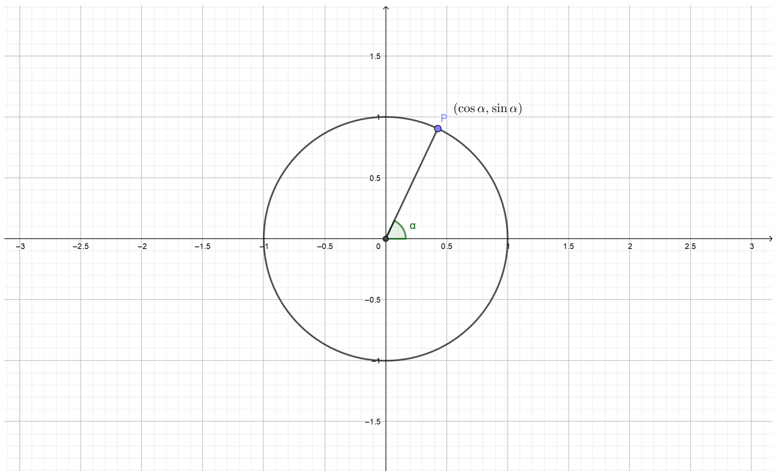
12. (10 points) A farmer wants to build two adjacent animal pens using 5 straight sections of fence, as shown below. Which dimensions will produce the **largest total area** if the farmer has 60 feet of fence to use?

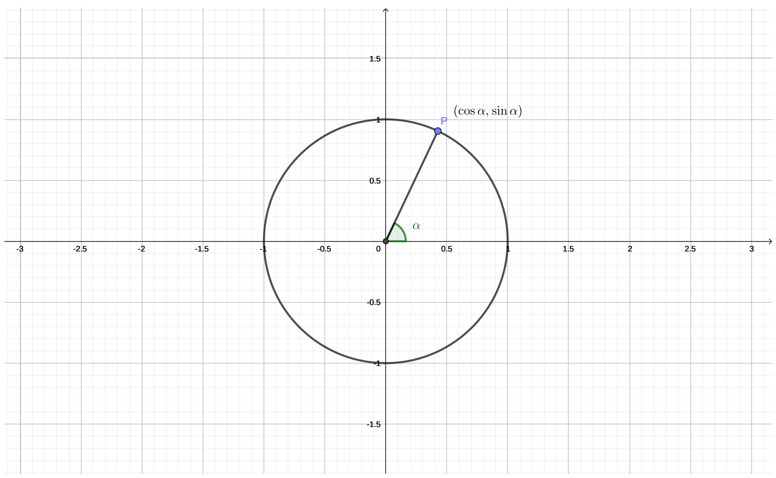


13. (10 points) Find the area of the region enclosed by $y = x^2 + 2x - 7$ and $y = x - 1$.



14. LaTeX can display saved images in formats like JPG, PNG, PDF, and more.





15. Here are a few more sample math expressions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\int_a^b f(x)\ dx = \lim_{n \to \infty} \left(\sum_{i=1}^{n} f(x_i^*) \Delta x_i \right)$$

$$\frac{d}{dx} \left[\int_a^x f'(t)\ dt \right] = f(x)$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\vec{v} = \langle 2, 3, -1 \rangle = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$$

$$\overline{AB}, \overleftrightarrow{AB}, \angle ABC, \measuredangle ABC$$

$$\triangle ABC \cong \triangle DEF$$

$$\sim (p \vee q) \iff \sim p \wedge \sim q$$