

---

# AN INITIAL-BOUNDARY VALUE PROBLEM FOR A DEGENERATE PARTIAL DIFFERENTIAL EQUATION INCLUDING TEMPERED CAPUTO FRACTIONAL OPERATOR

---

**Karimov Erkinjon**

Department of Mathematics and Informatics  
 Fergana State University  
 Fergana, Uzbekistan  
 erkinjon@gmail.com

**Murolimova Nargiza**

Department of Mathematics and Informatics  
 Fergana State University  
 Fergana, Uzbekistan  
 f.nargiza97@gmail.com

**Usmonov Doniyor**

Department of Mathematics and Informatics  
 Fergana State University  
 Fergana, Uzbekistan  
 dusmonov909@gmail.com

## Abstract

The present research is devoted to studying a unique solvability of an initial-boundary problem for a degenerate partial differential equation involving tempered Caputo fractional derivative in the time variable. First, we solve the Cauchy problem for an ordinary fractional differential equation with the tempered Caputo derivative representing its solution via the Kilbas-Saigo function. The solution of the main problem is represented in series form and using the explicit form of the corresponding Cauchy problem, certain properties of the Kilbas-Saigo function, and imposing certain conditions on given functions, we have proved the existence of a unique solution.

**Keywords:** Initial-boundary problem; Caputo tempered fractional derivative; degenerate differential equation; Kilbas-Saigo function.

**MSC 2020:** 35R11

## 1. Introduction

Text of the introduction.

For the cite references, please use [3].

## 2. Main part

For definitions, remarks, propositions, theorems, lemmas, and corollaries, please use the following formats:

**Definition 2.1.** Text of the definition.

**Remark 2.2.** Text of the remark.

**Proposition 2.3.** *Text of the proposition.*

**Proof.** The proof of the proposition.

The proof of Proposition 2.3 is complete.  $\square$

**Theorem 2.4.** *Text of the theorem.*

**Proof.** The proof of the theorem.

The proof of Theorem 2.4 is complete.  $\square$

**Lemma 2.5.** *Text of the lemma.*

**Proof.** The proof of the lemma.

Lemma 2.5 has been proved. □

**Corollary 2.6.** *Text of the corollary.*

**Proof.** The proof of the corollary.

Corollary 2.6 has been proved. □

For each group of objects, use its own numbers. (e.g. Theorem 1.1, Theorem 1.2,... Lemma 1.1, Lemma 1.2, ... Corollary 1.1, Corollary 1.2,...)

**Use double dollars for separate formulas:**

$$u_{xx} - u_t = f(x, t), \quad (x, t) \in \Omega_1.$$

**The numeration of formulas are as follows:**

$$u(x, 0) = f_1(x), \quad x \in \mathbb{R}, \quad (1)$$

where  $f_1(x)$  is a given function.

**For citing formulas please use the following format:**

Using the condition (1), we get...

**For tabular, please use the following format:**

Number of the type changing lines	Number of gluing conditions	Remarks
1	2	According to the order of the equation the number of gluing conditions change
2	4	According to the order of the equation the number of gluing conditions change

For the references use the following formats:

## References

1. Authors. Title of the book. Year of publication. City: Publisher.
2. Authors. Title of the article. The name of the Journal. Year. Vol.5, Issue 1, pp. 72-78.
3. Authors. Title of the abstract. The name of the Conference. City, Country. Year, pp. 1009-1015.
4. Authors. arXiv preprint. arXiv:1305.4992. Year.

TEMPERLANGAN KAPUTO KASR TARTIBLI HOSILA ISHTIROK ETGAN XUSUSIY HOSILALI TENGLAMA UCHUN  
 BOSHLANG‘ICH-CHEGARAVIY MASALA  
**Karimov Erkinjon, Murolimova Nargiza, Usmonov Doniyor**

Ushbu tadqiqot vaqt bo'yicha temperlangan Kaputo kasr tartibli hosilasi ishtirok etgan xususiy hosilali tenglama uchun bir boshlang'ich-chegaraviy masalaning bir qiymatli yechilishiga bag'ishlangan. Dastlab temperlangan Kaputo kasr tartibli hosilasi ishtirok etgan oddiy differential tenglama uchun Koshi masalasini yechamiz va yechimini Kilbas-Saygo funksiyasi orqali ifodalaymiz. So'ngra asosiy masala yechimini qatro ko'rinishida ifodalaymiz va Koshi masalasi yechimi, Kilbas-Saygo funksiyasi xossalardan foydalanib, berilganlarga ma'lum shartlar asosida tadqiq etilayotgan masalaning bir qiymatli yechilishini isbotlaymiz.

**Kalit so'zlar:** Boshlang'ich-chegaraviy masala; temperlangan Kaputo kasr tartibli hosilasi; buziladigan differentials tenglama; Kilbas-Saygo funksiyasi.

НАЧАЛЬНО-КРАЕВАЯ ЗАДАЧА ДЛЯ ВЫРОЖДАЮЩЕГОСЯ УРАВНЕНИЯ В ЧАСТНЫХ ПРОИЗВОДНЫХ С  
 ТЕМПЕРИРОВАННОЙ ДРОБНОЙ ПРОИЗВОДНОЙ КАПУТО  
**Каримов Эркинжон, Муролимова Наргиза, Усмонов Дониер**

Настоящее исследование посвящено однозначной разрешимости одной начально-краевой задачи для вырождающегося уравнения в частных производных с темперированной дробной производной Капуто по временной переменной. Сначала мы решаем задачу Коши для обыкновенного дифференциального уравнения с темперированной дробной производной Капуто представляя решение функцией Килбас-Сайго. Решение основной задачи представлено в виде ряда и пользуясь решением задачи Коши, а также свойствами функции Килбас-Сайго, при определенных условиях на заданные, доказано однозначная разрешимость исследуемой задачи.

**Ключевые слова:** Начально-краевая задача; темперированная дробная производная Капуто; вырождающееся дифференциальное уравнение; функция Килбас-Сайго.

**Received:** \*\*/\*\*/202\*

**Accepted:** \*\*/\*\*/202\*

### Cite this article

Karimov E., Murolimova N. and Usmonov D. An initial-boundary value problem for a degenerate partial differential equation including tempered Caputo fractional operator. *Bull. Inst. Math.*, 2023, Vol.6, No 1, pp. 1-3