Solving Force Problems - A Primer

Solving the physics problems you are presented with in an introductory course become easier if each time you sit down to solve a problem, you follow a generalized problem-solving strategy that includes sketching pictures.

You may find following the following procedure to solve force problems worthwhile:

1. Read the problem, identifying any numbers, paying close attention to the associated units with the consideration of converting each to a common unit system, e.g. kg, m, and s.
2. Identify the question(s) being asked.
3. Analyze the problem at hand using sketches on paper to decode the scenario.
4. Identify the principal item/object that is being "pushed" around, and remove it from your previous sketch in an isolated diagram (Free Body Diagram), identifying the pushes and pulls (forces) on it using directed line segments (vectors) which are best expressed as arrows (having tails and heads whose directions distinguish the asserted motions).
5. Ask yourself: is there an object that has a weight, or a a mass? Convert any masses to kilograms (kg), but recognize that mass and weight are related, but not identical, that is, the weight of an object W is given by $W = mg$, where m is its mass (in kg) and g is the acceleration that the Earth imposes on it (assuming you are on the Earth), most often referenced as the acceleration due to gravity having a value of $g = 9.8\ m/s^2$, which is close enough to $g = 10\ m/s^2$. You might prefer $g = 32\ ft/s^2$, noting this is equivalent to $g = 22\ mi/hr\ per\ second$, that is if an object is dropped from rest at a sufficiently high height, each second that it is in flight its velocity increases by 22 mi/hr owing to the Earth's gravitational influence on it.
6. Ask yourself: are there any tension forces involving ropes, strings, identify these with a variable T, and if multiple use T_1, T_2, etc.
7. Ask yourself: are there any forces imposed on the object due to any interacting surfaces, identifying these as Normal Forces (N), noting a "Normal Force" in math-speak references an orthogonal force, that is one that imposes a force on an object at a direction of 90 degrees. An example is an upward normal force balancing the weight of a book resting on a table.
8. Ask yourself: are there an frictional forces, identifying these with f, noting that frictional forces and normal forces are related as $f \leq \mu N$ where the

amount of resistance imposed by a frictional force is scaled as a constant μ which is generally a coefficient that spans $0 \leq \mu \leq 1$, where a $\mu = 0.01$ would be representative of a very slick situation, say a steel hockey skate blade on smooth ice, or $\mu = 1.0$ suggestive of say a rubbery tennis shoe on dry concrete.

9. Ask yourself: are there any other forces involved, maybe gravitational forces, maybe some specific push or pull, identifying these appropriately. Gravitational forces between two objects the one having a mass M, the other m, and separated by a distance of r, could be identified as $F_G = GMm/r^2$, where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

10. Select an orientation for a Cartesian Coordinate system that lays over your Free Body Diagram that allows as many of your vectors to lie on the $x-$ and/or $y-$axes (thus minimizing the pain/joys of using trigonometry). If the motion is restricted to one-dimension this is easier than otherwise, for example a person being pushed along the ground versus a person sliding down a ramp.

11. Inventory your forces on the Force Diagram, aligning the vectors tail to the origin of the coordinate axes, that is at the point $(0, 0)$, and its head pointing in the appropriate direction. Ideally each of the lengths of the vectors are scaled accordingly, for example if a weight vector is 200 N, a push or pull of 100 N would be drawn half as long.

12. Ask yourself: Is this a static force problem, or is it a dynamic force problem?

13. After your completed inventory of forces, use Newton's Second Law of Forces to engineer the relationships between forces. Newton's 2^{nd} is written $\vec{F}_{net} = m\vec{a}$, emphasizing that $\vec{F}_{net}$ and $\vec{a}$ are in fact vector quantities that require your identification of magnitude and direction, hopefully with lengths and directions drawn accurately on your Force Diagram.

14. Solve the equation(s) generated by Newton's 2^{nd} Law in variable format first, then plug in the numbers. Be sure you've converted all the variables to be consistent, generally best if expressed in kg, m, s.

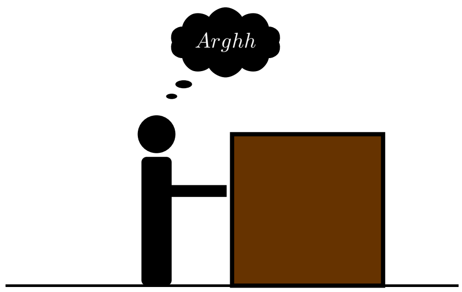
15. Ask yourself: Did you answer the question, does the result makes sense? If not, provide some explanation.

With the overview above, let's try solving a force problem.

Question: Suppose you have a $100 - kg$ wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be? (c) What would the crate's acceleration become if pushed up a wooden ramp with a force of 625 N that is oriented at an angle of 20^o relative to what would of been the horizontal direction?

This problem has three parts, let's work on this by doing part a), b), then c).

A sketch of the situation might look like:



Inventorying this situation, only the mass of the crate was given as $m = 100\ kg$, however having read the textbook to learn something about friction, here we have wood rubbing on wood, which upon inspection the coefficient of static friction $\mu_s = 0.5$ and for kinetic (sliding) friction $\mu_k = 0.3$. That is, given in the problem is m and μ.

The question being asked, somewhat translated is upon pushing the crate, at what point does is start to slip, that is how much effort do I need to apply to get it moving. What we want to find then is the applied force that is exerted, let's call that F_{push}, you might consider it as your a target variable.

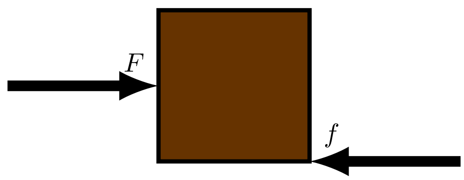
Besides F_{push}, there are some other forces involved. A question you can ask yourself is, "If I were the crate, what would I feel?" - Hmmm, I'd feel somebody pushing on me from the left, okay we called that F_{push}, I'd feel the Earth pulling on me (W for my weight, the result of two masses pulling on each other at a distance), I'd feel the floor pushing up on me (N for the perpendicular (normal) force imposed on me by the floor), and when I'm dragging along the floor I'd feel the friction imposed on me when my wooden base drags along the wooden floor, ack, that might hurt, do you have a can of oil that I can be squirted between me and the floor, it might keep the rashing down, okay guess not.

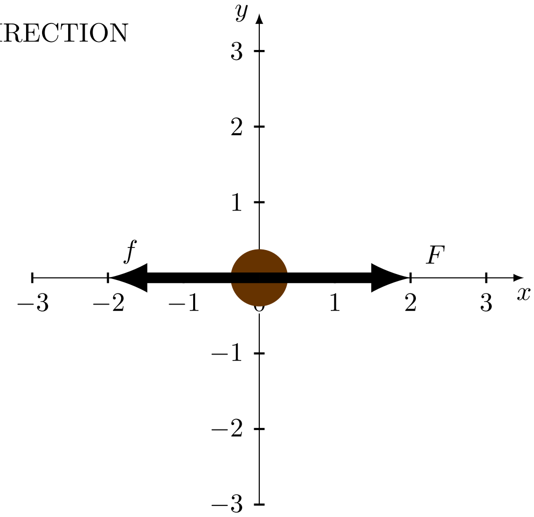
So all together, let's see, I have a F_{push}, a W, an N, a f, let's see, that should do it, let's cast those onto the so-called Free Body and Force Diagrams, but in two steps, first the forces that operate in the horizontal direction (F_{push} & f), and separately those operating in the vertical direction (W & N).

Editor's Note: The four vectors used below are drawn with equal proportion which is likely not the case. For the solution to Part a) of this problem it is true the applied force is balanced by the opposing frictional force, and that the Weight is balanced by the normal force, but drawing the four forces with equal proportions is hopefully not misleading.

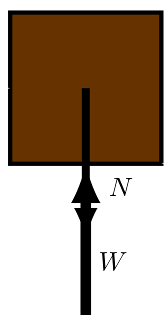
FREE-BODY DIAGRAM FORCE DIAGRAM

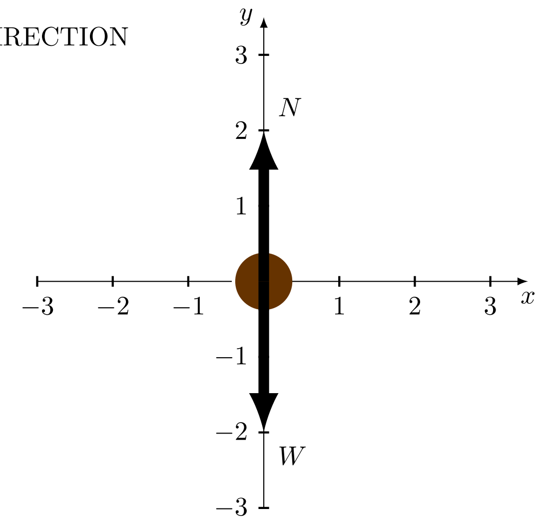
HORIZONTAL DIRECTION



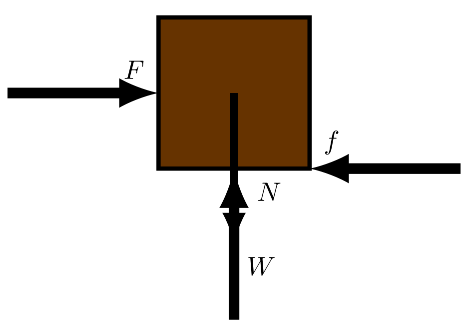


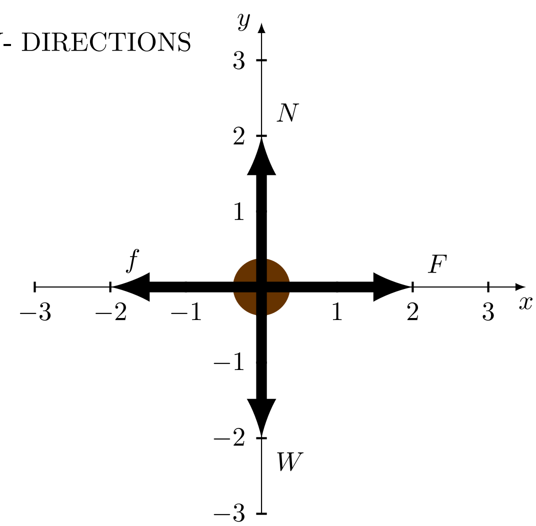
VERTICAL DIRECTION





COMBINED X- & Y- DIRECTIONS





FREE-BODY DIAGRAM FORCE DIAGRAM

Having identified the relevant forces, representing each on a Force Diagram, then casting those onto a 2D Cartesian Coordinate system, applying Newton's Law is straightforward:

$$\vec{F}_{net} = m\vec{a} \longrightarrow \begin{cases} x: F_x^{net} = ma_x \\ y: F_y^{net} = ma_y \end{cases}$$

That is, to use Newton's 2^{nd}, we can analyze the motion separately in each of the x and y dimensions.

"Adding up" the horizontal forces, results in: $F_x^{net} = F_{push} - f_{static} = ma_x$, noting that for the case where the crate does not slide, we identify the friction as static, and therefore the acceleration $a_x = 0$, generating the relationship $F_{push} - f_s = 0$ or that $F_{push} = f_s$, to interpret: the two forces balance each other in the case that there is no acceleration.

To determine the amount of effort F_{push} to overcome f_s, we need to know something about the frictional force f_s. What we've learned about friction is that it is the result of two surfaces rubbing on each other, and that friction is proportional to the perpendicular force imposed on the object at the interface of the two surfaces, that is, $f \leq \mu N$ where the proportionality constant μ (aka coefficient of friction) has the property $0 \leq \mu \leq 1$ as described previously. Somewhat peculiar is that this is expressed as a range ($\leq N$), this topic being addressed in the textbook.

However, we seem to be kicking the can down the road, to get F_{push} we need f, but that requires μ, which we know, but also N. We can generate a relationship between N and W by studying the Force Diagram but in the vertical direction. There we see "adding up" the vertical forces results in $F_y^{net} = +N - W = ma_y$. Since the crate is not in vertical motion while being pushed horizontally, its acceleration $a_y = 0$, therefore $N - W = 0$ or that $N = W$, and you recall that $W = mg$, so geez, that's one can kicking exercise.

To vertically rehash: $N = mg$, then $f_s = \mu N = \mu mg$ and to horizontally rehash:

$$F_{push} = f_s = \mu mg = 0.5 \times 100\ kg \times 10\ m/s^2 = 500\ kg\ m/s^2 = 500\ N$$

Does this solution make sense? If I convert Newtons to Pounds, hmm, let's see:

$$500\ N = \left(\frac{4.448\ lb}{1\ N}\right) = 112.4\ lbs$$

Hmm, okay it will take a force of 112 *lbs* to get the crate moving, okay, I might be able to apply that if I push hard enough. But what is interesting, if I first apply say 20 *lbs* of effort to the right, no motion, then 40 *lbs*, no motion, 60 *lbs*, no motion, 80, 100, 110 *lbs*, nothing, but at 112 *lbs* of effort, the wooden crate will break free on the wooden floor and slide. Note that it did not matter on how much of the area of the crate was in contact with the floor, that is this would work if the crate was tiny, or huge as along as it's mass were 100 *kg*.

Editor's Note: Okay, we solved Part a), now onto Part b.

For Part b we are asked to continue to apply the $F_{push} = 500\ N$, okay maybe just a nudge more, which sets the crate into motion, at what rate does the velocity increase with time, that is what is the resulting acceleration?

Inventorying what we have: KNOWN: $m = 100\ kg$, $F_{push} = 500\ N$, and $\mu_k = 0.3$ since the wood crate-on-wood floor is in motion.

We can reference the same Free-body and Force Diagrams, and use Newton's Second Law to generate two relationships, one horizontally, and one vertically: $F_x^{net} = F_{push} - f_k = ma_x$ and $F_y^{net} = N - W = 0$, the second resulting in the recognition that the case is not hopping up and down in the horizontally applied push (motion).

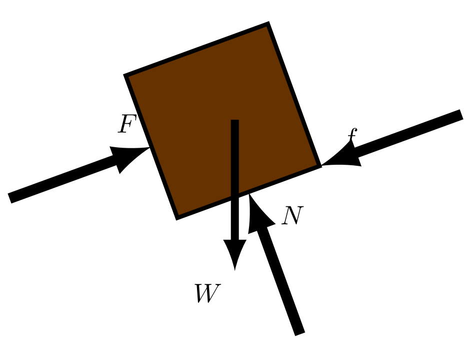
Maneuvering with $f_k = \mu_k N$, we then have $F_{push} - \mu_k N = ma_x$ or:

$$a_x = \frac{F_{push} - \mu_k W}{m} = \frac{F_{push} - \mu_k mg}{m} = \frac{500 - (0.3 \times 100 \times 10)}{100} = 2\ m/s^2$$

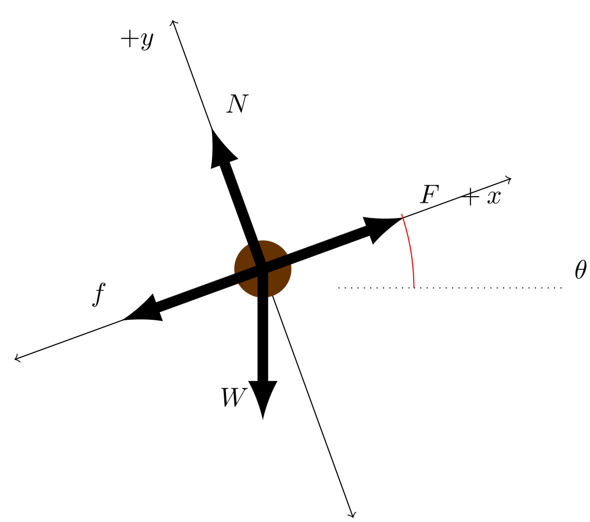
Is this a sensible solution? Suggesting so if you recognize that it's one fifth that of the acceleration of gravity, and that it would be possible for a person to speed up at that rate.

For part C, things get a bit more complicated because of the ramp oriented at twenty degrees. The problem is that if I were to push that wooden crate up a wooden ramp with an increased force of $F_{push} = 625\ N$, what would the resulting acceleration be? From the onset, I'd guess that it would be less than the 2 m/s^2 that I discovered in the Part b. But, let's try first by drawing the Free-Body and Force Diagrams for this situation:

FREE-BODY DIAGRAM



FORCE DIAGRAM



Inventorying what we KNOW: F_{push}, m, μ_k, and $\theta = 20^o$, with the burning question: What is the resulting acceleration a_x?

Because the motion remains in one-dimension relative to the ramp, it behooves us to tilt the coordinate axes to lie along the direction of motion.

With well done Diagrams, applying Newton's Second should be coming more and more systematic for you:

$$F_x^{net} = F_{push} - f_k - W_x = ma_x$$

where $W_x = W sin\theta$, noting that the angle between the skewed $y - axis$ and $\vec{W}$ is θ - please try sketching the geometry of the situation to prove it to yourself.

$$F_y^{net} = N - W cos\theta = ma_y = 0$$

or that $N = mgcos\theta$, since the crate will not be in motion vertically because of the selection of the orientation of the $x - axis$.

We are after the acceleration a_x where:

$$a_x = \frac{F_{push} - \mu_k(mgcos\theta) - mgsin\theta}{m}$$

$$a_x = \frac{625 - (0.3 \times 100 \times 10 \times cos(20)) - (100 \times 10 \times sin(20))}{100} = 0.02\ m/s^2$$

That is a number that is pretty close to zero, thus the applied force just barely is enough to keep the crate from sliding backwards onto the pusher.

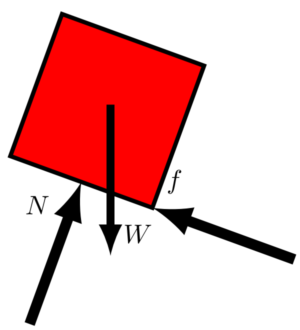
I challenge the reader with the following problem:

Determine the coefficient of friction that would correspond to a $100-kg$ wooden crate that started slipping when the ramp was adjusted to $20-degrees$ relative to the horizontal.

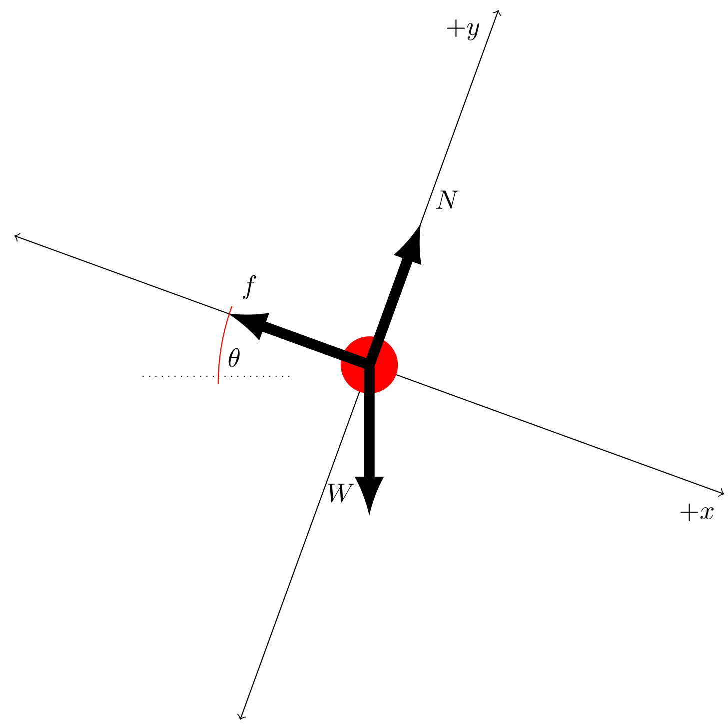
For convenience, I provide the associated Free-body and Force Diagrams.

A complete solution submitted to your instructor will result in an extra credit score of as many as 20 points.

FREE-BODY DIAGRAM



FORCE DIAGRAM



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