A Derivation of the Three-dimenional Pythagorean Theorem

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Abstract

This short paper deals with a derivation of the three-dimensional Pythagorean Theorem employing the general equation of a circle. However, I am not presenting this paper with any kind of proofs or explanations for the concepts assiting the derivation of the Pythagorean Theorem.

1 The Proof

A circle is the graph of all those points whose distance from a specific point is the same. That distance is called radius, which is the length of the line segment, called the radial segment, joining any of those points to the specific point. The equation fulfilling these conditions can be given thus

$$r = \sqrt{x^2 + y^2} \tag{1}$$

This is precisely the equation of any circle placed at the origin of a two-dimensional coordinate system.

The three dimensional pythagorean theorem can be given thus

$$\alpha = \sqrt{x^2 + y^2 + z^2} \tag{2}$$

where α is the distance of any point from the origin of a three dimensional coordinate system, and x, y, z are the coordinates of that point in the same coordinate system.

Now we set out to derive the three-dimensional Pythagorean Theroem.

Consider a circle, let us say A with its centre placed at the origin of a three-dimensional coordinate system. The circle A has the radius r. We also specify

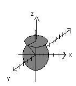

Figure 1: The circle A and B in the 3D-coordinate system.

that one of the diameters of circle A is parallel to the z-axis. Now, consider another circle, let us say circle B, and its centre is placed at the point $(0, 0, r)$. Two of the infinitely many diameters of circle B are parallel to the x and y-axes. Circle B has the radius r_1. We know that the distance of any point on the circumference of circle B can be given thus

$$p = \sqrt{r^2 + r_1^2} \tag{3}$$

We also know that the distance of any point on the circumference of circle B from the point $(0, 0, r)$, where $(0, 0, r)$ can be treated as the origin, can be given thus

$$q = sqrtx^2 + y^2 \tag{4}$$

where x and y are the coordinate of any point on the circumference of circle B.

We know that $r_1 = q$, hence from equation (3)

$$r_1 = sqrtp^2 - r^2 \tag{5}$$

and therefore,

$$\sqrt{p^2 - r^2} = \sqrt{x^2 + y^2} \tag{6}$$

$$p^2 = x^2 + y^2 + r^2 \tag{7}$$

$$p = sqrtx^2 + y^2 + r^2 \tag{8}$$

Note that in the point (0,0,r), r was the z-coordinate of point where it exists the centre of circle B. The numerical value of r as a coordinate is the same as r where it is the radius of the circle A. Thus, the three- dimensional can be derived.